1. Achromatic Doublet **(5 pts)**

Design an achromatic doublet for a nominal focal length $f = 200$ mm for the center wavelength of the Na D doublet ($\lambda = 589.3$ nm) from 518:596 crown glass ($n = 1.518$, $V = 59.6$) and 617:366 dense flint glass ($n = 1.617$, $V = 36.6$). A positive lens is followed by a negative lens whose concave front face fits the convex back face of the L^+. What are the curvature radii $R_1 = -R_2 = -R_3$ and R_4 of the lenses to obtain the target focal length of the pair?

With $\rho_1 = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$; $\rho_2 = \left(\frac{1}{R_2} - \frac{1}{R_3}\right)$ and $\frac{1}{f} = (n_1 - 1)\rho_1 + (n_2 - 1)\rho_2$ we obtain

$\frac{\rho_1}{\rho_2} = -\frac{n_{2blu} - n_{2red}}{n_{1blu} - n_{1red}} = \frac{(n_{2ylw} - 1) f_{2ylw}}{(n_{1ylw} - 1) f_{1ylw}}$ by requiring $f_{red} = f_{blu}$, and therefore,

$$\frac{f_{2ylw}}{f_{1ylw}} = -\frac{n_{2blu} - n_{2red}}{n_{2ylw} - 1} \Big/ \frac{n_{1blu} - n_{1red}}{n_{1ylw} - 1} = -V_2/V_1$$

Hence, $f_{1ylw} = f\left(1 - V_2/V_1\right) = 200\text{ mm} \cdot \left(1 - 366/596\right) = 77.2$ mm and

$f_{2ylw} = f\left(1 - V_1/V_2\right) = 200\text{ mm} \cdot \left(1 - 596/366\right) = -125.7$ mm

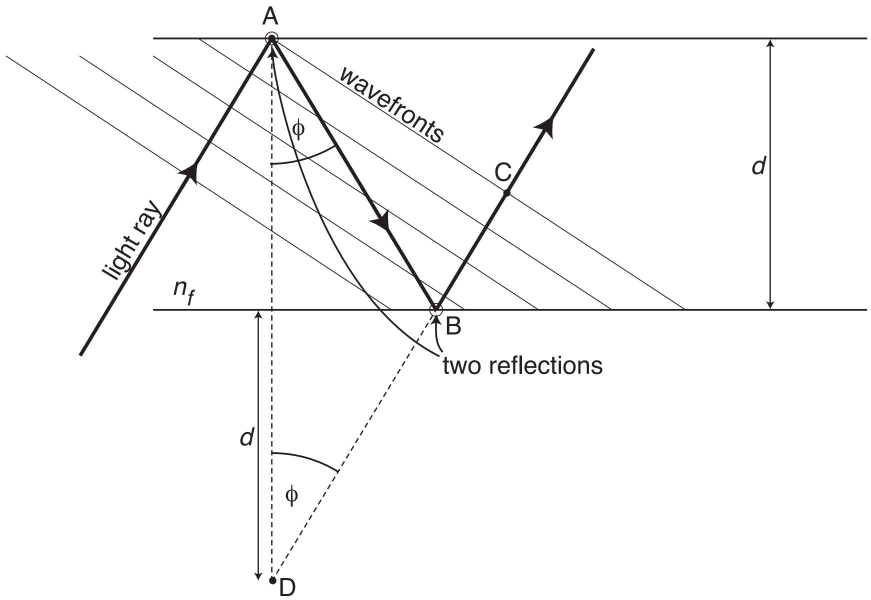
From $\frac{1}{f_1} = (n_{crown} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n_{crown} - 1) \cdot \frac{2}{R_1}$ follows $R_1 = 2 \cdot 77.2 \text{ mm} \cdot 0.518 = 80.0$ mm and

from $\frac{1}{f_2} = (n_{flint} - 1)\left(\frac{1}{R_3} - \frac{1}{R_4}\right) = (n_{flint} - 1)\left(-\frac{1}{R_1} - \frac{1}{R_4}\right)$ follows

$R_4 = -(n-1)\frac{f \cdot R_1}{R_1 + (n-1) f} = 2607.5$ mm (remember that $f_2 < 0$!)

2. Propagated Modes in Optical Fiber **(4 pts)**

(a) Explain qualitatively, by sketching the geometry of a planar light guide, why there is a selection of transmitted modes in the angular directions of rays within the light guide.



Phase of wave at point **A** must match phase after two reflections at point **C**.

(b) Using the geometric argument from the sketch, derive the resonance condition that determines the distribution of modes.

Optical path {ABC} equals {DC}, $\overline{\mathrm{DC}} = 2d\cos\phi$

→ resonance condition:
$m\lambda = 2dn_f \cos\phi + 2\phi_r$ where ϕ_r is phase jump upon reflection.

3. Standing Wave I **(4 pts)**

Use the complex representation to find the composite wave $E = E_1 + E_2$ that derives from

$E_1(x,t) = E_0 \cos(kx - \omega t)$ and $E_2(x,t) = -E_0 \cos(kx + \omega t)$.

$$E(x,t) = E_0\left(\mathrm{e}^{i(kx-\omega t)} - \mathrm{e}^{i(kx+\omega t)}\right) = E_0\mathrm{e}^{ikx}\left(\mathrm{e}^{-i\omega t} - \mathrm{e}^{i\omega t}\right) = -2iE_0\mathrm{e}^{ikx}\sin\omega t$$

Remember $\mathrm{e}^{ix} = \cos x + i\sin x$ to obtain $E(x,t) = -2iE_0(\cos kx \cdot \sin\omega t + i\sin kx \cdot \sin\omega t)$

The real part of this is ($-i^2 = 1$): $E(x,t) = 2E_0 \sin kx \cdot \sin\omega t$

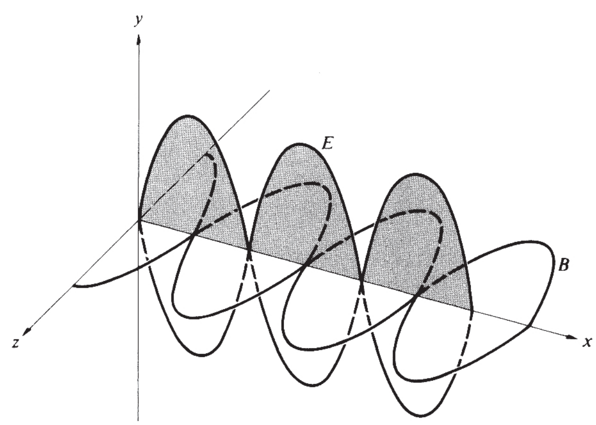
4. Standing Wave II **(4 pts)**

The solution of problem 4 is the standing wave, $\vec{E}(x,t) = 2\vec{E}_0 \sin kx \cdot \sin \omega t$ where $\vec{E}_0$ points in the direction of $\hat{e}_y$. Derive the field $\vec{B}$ associated with $\vec{E}$ and draw a sketch of the two fields over one wavelength λ.

From $\partial E/\partial x = -\partial B/\partial t$ it follows that

$$B(x,t) = -\int \frac{\partial E}{\partial x}\mathrm{d}t = 2E_0 k\cos(kx)\int \sin(\omega t)\mathrm{d}t = -2E_0 k/\omega\cos(kx)\cos(\omega t) \text{ and with } \frac{E_0 k}{\omega} = \frac{E_0}{c} = B_0$$

$$B(x,t) = -2B_0\cos(kx)\cos(\omega t)$$



5. Dispersion Relation in Plasma **(3 pts)**

A plasma is a dispersive medium for EM waves with a dispersion relation described by

$\omega^2 = \omega_p^2 + c^2k^2$ where ω_p is the plasma frequency. Derive the phase and group velocities, v_{ph} and v_g, in the plasma as functions of ω and ω_p and show that $v_{ph} \cdot v_g = c^2$.

$$\omega^2 = \omega_p^2 + c^2k^2 \quad \rightarrow \quad c^2k^2 = \omega^2 - \omega_p^2; \quad \frac{ck}{\omega} = \sqrt{1-\omega_p^2/\omega^2}$$

$$v_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{1-\omega_p^2/\omega^2}}, \; v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = 2c^2k\cdot\frac{1}{2}\left(\omega_p^2 + c^2k^2\right)^{-1/2} = \frac{c^2k}{\omega} = c\cdot\sqrt{1-\omega_p^2/\omega^2}$$

$v_{ph} \cdot v_g = c^2$